NUCLEON SUM RULES IN SYMMETRIC AND ASYMMETRIC NUCLEAR MATTER *

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Abstract

We calculate the nucleon parameters in isospin symmetric and asymmetric nuclear matter using the QCD sum rules. The higher moments of the nucleon structure functions are included. The complete set of the nucleon expectation values of the four-quark operators is employed. We analyze the role of the lowest order radiative corrections beyond the logarithmic approximation.

1 Introduction

We investigate the vector and scalar self-energies of nucleons in nuclear matter composed by the neutrons and protons, distributed with densities ρ_n and ρ_p. We calculate the dependence on the total density $\rho = \rho_p + \rho_n$ and on the asymmetry parameter $\beta = (\rho_n - \rho_p)/(\rho_p + \rho_n)$.

The QCD sum rules were invented in paper [1] to express the hadron parameters through the vacuum expectation values of QCD operators. Being initially used for the mesons, the method was expanded in [2] to the description of the baryons. The approach succeeded in describing the static characteristics as well as some of the dynamical characteristics of the hadrons in vacuum — see, *e.g.* the reviews [3].

Later the QCD sum rules were applied for investigation of modified nucleon parameters in the symmetric nuclear matter [4]. They were based on the Borel-transformed dispersion relation for the function $\Pi_m(q)$ describing the propagation

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of the system with the quantum numbers of the nucleon (the proton) in the nuclear matter. Considering nuclear matter as a system of A nucleons with momenta p_i, one introduces the vector $p = (\Sigma p_i)/A$, which is thus $p \approx (m, 0)$ in the rest frame of the matter. The function $\Pi_m(q)$ can be presented as $\Pi_m(q) = \Pi_m(q^2, \varphi(p, q))$ with the arbitrary function $\varphi(p, q)$ being kept constant in the dispersion relations in q^2.

The general form of the function Π_m can thus be presented as

$$\Pi_m(q) \;=\; q_\mu \gamma^\mu \Pi_m^q(q^2, s) + p_\mu \gamma^\mu \Pi_m^p(q^2, s) + I \Pi_m^I(q^2, s) \;. \tag{1}$$

The in-medium QCD sum rules are the Borel-transformed dispersion relations for the components $\Pi_m^j(q^2, s)$ $(j = q, p, I)$

$$\Pi_m^j(q^2, s) \;=\; \frac{1}{\pi} \int \frac{\text{Im } \Pi_m^j(k^2, s)}{k^2 - q^2}\, dk^2 \;. \tag{2}$$

The spectrum of the function $\Pi_m(q)$ is much more complicated than that of the function $\Pi_0(q)$ describing the propagation of the system with the quantum numbers of the nucleon in the vacuum. The choice of the function $\varphi(p, q)$ is dictated by the attempts to separate the singularities connected with the nucleon in the matter from those connected with the properties of the matter itself. Since the latter manifest themselves as singularities in the variable $s = (p + q)^2$, the separation can be done by putting $\varphi(p, q) = (p + q)^2$ and by fixing [5] $\varphi(p, q) = (p + q)^2 \equiv s = 4m^2$ (m is the nucleon mass).

By using Eq. (2) the characteristics of the nucleon in nuclear matter can be expressed through the in-medium values of QCD condensate. The possibility of extension of "pole + continuum" model [1, 2] to the case of finite densities was shown in [5]–[7].

The lowest order of OPE of the lhs of Eq. (2) can be presented in terms of the vector and scalar condensates [5], [7]. Vector condensates $v_\mu^i = \langle M|\bar{q}^i \gamma_\mu q^i|M\rangle$ of the quarks with the flavor i ($|M\rangle$ denotes the ground state of the matter) are the linear functions of the nucleon densities ρ_n and ρ_p. In the asymmetric matter both SU(2) symmetric and asymmetric condensates

$$v_\mu = \langle M|\bar{u}(0)\gamma_\mu u(0) + \bar{d}(0)\gamma_\mu d(0)|M\rangle = v_\mu^u + v_\mu^d$$

$$v_\mu^{(-)} = \langle M|\bar{u}(0)\gamma_\mu u(0) - \bar{d}(0)\gamma_\mu d(0)|M\rangle = v_\mu^u - v_\mu^d$$

obtain nonzero values. In the rest frame of the matter $v_\mu^i = v^i \delta_{\mu 0}$, $v_\mu = v\delta_{\mu 0}$, $v_\mu^{(-)} = v^{(-)}\delta_{\mu 0}$. We can present $v^i = \langle n|\bar{q}^i \gamma_0 q^i|n\rangle \rho_n + \langle p|\bar{q}^i \gamma_0 q^i|p\rangle \rho_p$. The values $\langle N|\bar{q}^i \gamma_0 q^i|N\rangle$ are just the numbers of the valence quarks in the nucleons $\langle n|\bar{u}\gamma_0 u|n\rangle = \langle p|\bar{d}\gamma_0 d|p\rangle = 1$, $\langle p|\bar{u}\gamma_0 u|p\rangle = \langle n|\bar{d}\gamma_0 d|n\rangle = 2$, and thus

$$v^u = \rho_n + 2\rho_p = \rho\left(\frac{3}{2} - \frac{\beta}{2}\right), \quad v^d = 2\rho_n + \rho_p = \rho\left(\frac{3}{2} + \frac{\beta}{2}\right). \tag{3}$$

Hence, we obtain $v(\rho) = v_N\rho, v^{(-)}(\rho,\beta) = \beta v_N^{(-)}\rho$ with $v_N = 3$, $v_N^{(-)} = -1$.

The lhs of Eq. (2) contains the SU(2) symmetric scalar condensate $\kappa_m(\rho) = \langle M|\bar{u}(0)u(0)+\bar{d}(0)d(0)|M\rangle$, and SU(2) asymmetric one $\zeta_m(\rho,\beta) = \langle M|\bar{u}(0)u(0)-\bar{d}(0)d(0)|M\rangle$. These condensates can be presented as

$$\kappa_m(\rho) = \kappa_0 + \kappa(\rho)\ , \quad \kappa(\rho) = \kappa_N\rho + \dots\ , \quad \kappa_N = \langle N|\bar{u}u + \bar{d}d|N\rangle\ , \tag{4}$$

$\kappa_0 = \kappa_m(0)$ is the vacuum value, and

$$\zeta_m(\rho,\beta) \ = \ -\beta(\zeta_N\rho + \dots)\ , \quad \zeta_N = \langle p|\bar{u}u - \bar{d}d|p\rangle\ . \tag{5}$$

The dots in the rhs of Eqs. (4) and (5) denote the terms, which are nonlinear in ρ. In the gas approximation such terms should be omitted. The SU(2) invariance of vacuum was assumed in Eq. (5). The expectation value κ_N is related to the πN sigma term $\sigma_{\pi N}$ [8].

The gluon condensate

$$g_m(\rho) \ = \ \langle M|\frac{\alpha_s}{\pi}\, G^2(0)|M\rangle \ = \ g_0 + g(\rho) \ = g_0 \ + \ g_N\rho + \ \dots$$

$g_0 = g_m(0)$ is the vacuum value and $g_N = -\frac{8}{9}\,m$ obtained in a model-independent way.

We shall analyze the sum rules in the gas approximation. It is a reasonable starting point, since the nonlinear contributions to the most important scalar condensate $\kappa(\rho)$ are relatively small at the densities of the order of the phenomenological saturation density $\rho_0 = 0.17\,\mathrm{fm}^{-3}$ of the symmetric matter [7].

In the second part of the talk we discuss the role of the radiative corrections. The analysis [2] included also the most important radiative corrections, in which the coupling constant α_s is enhanced by the "large logarithm" $\ln q^2$. The corrections $(\alpha_s \ln q^2)^n$ have been included in to all orders for the leading OPE terms. This approach provided us with good results for the nucleon mass and for the other characteristics of nucleons.

However, inclusion of the lowest order radiative corrections beyond the logarithmic approximation made the situation somewhat more complicated. A numerically large coefficient of the lowest radiative correction to the leading OPE of the polarization operator $\Pi_0(q)$ was obtained in [9]. A more consistent calculation [9] provided this coefficient to be about 6. Thus, the radiative correction increases this term by about 50% at $|q^2| \sim 1\,\mathrm{GeV}^2$, which are actual for the SR analysis. This uncomfortably large correction is often claimed as the most weak point of the SR approach [9].

The radiative corrections of the order $\alpha_s \ln q^2$ and α_s for the contributions up to q^{-2} have been calculated in [10].

The further development of the nucleon SR in nuclear matter needs the calculation of the radiative corrections. This work is in progress and now we have present the analysis of the role of the radiative corrections in vacuum [11].

2 Sum rules in nuclear matter

We present the nucleon propagator in nuclear matter as

$$G_N^{-1} \ = \ q_\mu\gamma^\mu - m - \Sigma\,, \quad \Sigma \ = \ q_\mu\gamma^\mu\Sigma_q + p_\mu\gamma^\mu\frac{\Sigma_p}{m} + \Sigma_s \tag{6}$$

with the total self-energy Σ . We shall use the QCD sum rules for the calculation of the nucleon characteristics

$$\Sigma_v = \frac{\Sigma_p}{1-\Sigma_q}\ , \quad m^* = \frac{m+\Sigma_s}{1-\Sigma_q}\ , \quad \Sigma_s^* = m^* - m\ , \tag{7}$$

identified with the vector self-energy, Dirac effective mass and the effective scalar self-energy — see *e.g.* [12].

$$G_N \ = \ Z_N \cdot \frac{q_\mu\gamma^\mu - p_\mu\gamma^\mu(\Sigma_v/m) + m^*}{q^2 - m_m^2} \tag{8}$$

with Σ_v and m^* defined by Eq. (7). The new position of the nucleon pole is

$$m_m^2 \ = \ \frac{(s-m^2)\Sigma_v/m - \ \Sigma_v^2 + m^{*2}}{1+\Sigma_v/m}\ , \quad Z_N \ = \ \frac{1}{(1-\Sigma_q)(1+\Sigma_v/m)}\ . \tag{9}$$

We present also the result for the single-particle potential energies

$$U \ = \ \Sigma_s^* + \Sigma_v\ . \tag{10}$$

We trace the dependence of these characteristics on the total density ρ and on the asymmetry parameter.

The Borel-transformed sum rules take the form

$$L_m^q(M^2, W_m^2) \ = \ \Lambda_m(M^2)\ , \tag{11}$$
$$L_m^p(M^2, W_m^2) \ = \ -\Sigma_v\Lambda_m(M^2)\ , \tag{12}$$
$$L_m^I(M^2, W_m^2) \ = \ m^*\Lambda_m(M^2) \tag{13}$$

with $\Lambda_m(M^2) \ = \ \lambda_m^{*2}e^{-m_m^2/M^2}$, where λ_m^{*2} is the effective value of the nucleon residue in nuclear matter, M is Borel mass ($0.8 \leq M^2 \leq 1.4\mathrm{GeV}^2$).

We shall include subsequently the contributions of three types. The terms $\ell_m^j(M^2)$ ($j = p, q, I$) stand for the lowest order local condensates. These contributions are similar to simple exchanges by isovector vector and scalar mesons between the nucleon and the nucleons of the matter. The terms $u_m^j(M^2)$ are caused by the nonlocalities of the vector condensate. They correspond to the account of the form factors in the vertices between the isovector mesons couple to the nucleons. Finally, $\omega^j(M^2)$ describes the contributions of the four-quark condensates. They correspond to the two-meson exchanges (or to exchanges by

four-quark mesons, if there are any) and to somewhat more complicated structure of the meson-nucleon vertices. Thus we present the lhs of Eqs. (11)–(13) as

$$L^j_m \;=\; \ell^j_m + u^j_m + w^j_m. \tag{14}$$

Actually, we shall solve the sum rules equations, subtracting the vacuum effects [13] :

$$L^j \;=\; \ell^j + u^j + w^j \tag{15}$$

with $\ell^j = \ell^j_m - \ell^j_0$, $u^j = u^j_m$, $\omega^j = \omega^j_m - \omega^j_0$, while ℓ^j_0 and ω^j_0 are the corresponding contributions in the vacuum case.

A. Local condensates of the lowest dimensions.

The terms ℓ^i have the form:

$$\ell^q \;=\; f^q_v(M^2, W^2_m)v^q(\rho) + f^q_g(M^2, W^2_m)g(\rho), \tag{16}$$

$$\ell^p \;=\; f^p_v(M^2, W^2_m)v^p(\rho, \beta)\ , \tag{17}$$

$$\ell^I \;=\; f^I_\kappa(M^2, W^2_m)t^I(\rho, \beta)\ , \tag{18}$$

with the dependence on ρ and β being contained in the factors

$$v^q(\rho) = 3\rho, \quad v^p(\rho, \beta) = 3\rho\left(1 - \frac{\beta}{4}\right), \quad t^I(\rho, \beta) = \rho(\kappa_N + \zeta_N\beta), \tag{19}$$

and the function $g(\rho)$, given by Eqs. (4) and (5). The other functions are [14]

$$\begin{aligned}
f^q_v(M^2, W^2_m) &= -\frac{8\pi^2}{3}\frac{(s - m^2)M^2E_{0m} - M^4E_{1m}}{m\,L^{4/9}},\\
f^q_g(M^2, W^2_m) &= \frac{\pi^2M^2E_{0m}}{L^{4/9}},\\
f^p_v(M^2, W^2_m) &= -\frac{8\pi^2}{3}\ \frac{4M^4\,E_{1m}}{L^{4/9}}\ ,\\
f^I_\kappa(M^2, W^2_m) &= -4\pi^2M^4E_{1m}\ .
\end{aligned} \tag{20}$$

The notation $E_{km}\,(k = 0, 1)$ means that the functions $E(x) = 1 - e^{-x}\Sigma^n_{k=0}\frac{x^n}{n}$ depend on the ratio W^2_m/M^2. The factor $L = \ln(M^2/\Lambda^2)/\ln(\nu^2/\Lambda^2)$, where $\Lambda = 0.15$ GeV, $\nu = 0.5$ GeV.

B. Inclusion of the nonlocal condensates.

Finally, the higher moments and higher twists of the nucleon structure functions provide the contributions u^i to the L^i of the sum rules — Eq. (15)

$$\begin{aligned}
u^q(M^2) \;&=\; (u^q_{N,1}(M^2) + \beta u^q_{N,2}(M^2))\rho\ ,\\
u^q_{N,1}(M^2) &= \frac{8\pi^2}{3L^{4/9}m}\left[-\frac{5}{2}m^2M^2E_{0m}\langle\eta\alpha\rangle + \frac{3}{2}m^2(s - m^2)\langle\xi\rangle\right]\ ,
\end{aligned}$$

$$u^q_{N,2}(M^2) = \frac{8\pi^2}{3L^{4/9}m}\left[\frac{3}{2}\,m^2M^2E_{0m}(\langle\eta^u\alpha\rangle - \langle\eta^d\alpha\rangle)\right]; \tag{21}$$

$$\begin{aligned}
u^p(M^2) \; &= \; (u^p_{N,1}(M^2) + \beta u^p_{N,2}(M^2))\rho \; , \\
u^p_{N,1}(M^2) &= \frac{8\pi^2}{3L^{4/9}}\Big[-5\left(M^4E_{1m} - (s-m^2)M^2E_{0m}\right)\langle\eta\alpha\rangle \\
&- \frac{12}{5}m^2M^2E_{0m}\langle\eta\alpha^2\rangle + \frac{18}{5}m^2M^2E_{0m}\langle\xi\rangle\Big] \; , \\
u^p_{N,2}(M^2) &= \frac{8\pi^2}{3L^{4/9}}\Big[3\left(M^4E_{1m} - (s-m^2)M^2E_{0m}\right)\langle(\eta^u-\eta^d)\alpha\rangle \\
&+ \frac{9}{5}\,m^2M^2E_{0m}\langle(\eta^u-\eta^d)\alpha^2\rangle - \frac{27}{10}\,m^2M^2E_{0m}\langle(\xi^u-\xi^d)\rangle\Big];
\end{aligned} \tag{22}$$

$$u^I(M^2) \; = \; 0 \; . \tag{23}$$

Here we denote $\xi^u = -0.24$, $\xi^d = -0.09$ [15] and $\xi = \xi^u + \xi^d$. We use the structure functions η, obtained in [16] for the calculation of the terms u^q and u^p. Here we denote $\langle f\rangle = \int_0^1 d\alpha f(\alpha)$.

C. Inclusion of the four-quark condensates.

The calculations of the contributions of the four-quark condensates require the model assumption on the structure of the nucleon. The complete set of the four-quark condensate was obtained in [17] by using the perturbative chiral quark model (PCQM). There are three types of contributions to the four-quark condensate in the framework of this approach. All four operators can act on the valence quarks. Also, four operators can act on the pion. There is a possibility that two of the operators act on the valence quarks while the other two act on the pions. Using the complete set of the nucleon four-quark expectation values [17], we obtain

$$(\Pi)_{4q} \;\; = \;\; \left(A^q_{4q}(\beta)\frac{\hat q}{q^2} + A^p_{4q}(\beta)\frac{(pq)}{m^2}\frac{\hat p}{q^2} + A^I_{4q}(\beta)m\frac{I}{q^2}\right)\frac{a}{(2\pi)^2}\rho \tag{24}$$

with $a \; = \; -(2\pi)^2\langle 0|\bar u u|0\rangle$. The calculations give

$$A^q_{4q} = -0.11 - 0.21\beta, \quad A^p_{4q} = -0.57 + 0.09\beta, \quad A^I_{4q} = 1.90 - 0.92\beta \; . \tag{25}$$

The contributions of the four-quark condensates to the lhs of the Borel transformed sum rules (15) can be presented as

$$\begin{aligned}
&\omega^j = \omega^j_N\rho \; , \quad \omega^j_N = A^j_{4q}(\beta)f^j_{4q} \; , \\
&f^q_{4q} = -8\pi^2 a, \quad f^p_{4q} = -8\pi^2\frac{s-m^2}{2m}a, \quad f^I_{4q} = -8\pi^2 ma.
\end{aligned} \tag{26}$$

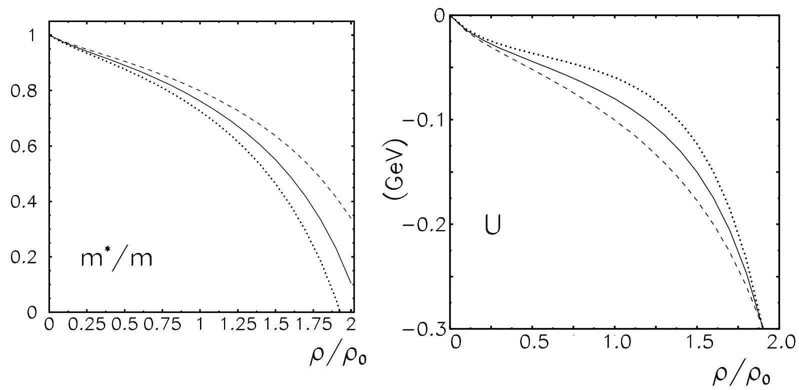

Figure 1: Results for m^* and $U(\rho)$ both in symmetric and neutron matter. Solid curves are for symmetric matter; proton (dashed) and neutron (dotted) curves are for the neutron matter.

In Fig. 1 we show results for effective nucleon mass m^* (7) and for the single-particle potential energies (10) in symmetric and asymmetric nuclear matter. During calculation we use the value $\kappa_N = 8$ (4), and $\zeta_N = 0.54$ (5) in Eq.(19) [13].

3 The role of radiative corrections

We have made [11] the analysis of radiative corrections to the nucleon SR in vacuum. The lowest OPE terms of the operators $\Pi_0^q(q^2)$ and $\Pi_0^I(q^2)$ in vacuum can be presented as

$$\Pi_0^q = A_0 + A_4 + A_6 + A_8; \quad \Pi_0^I = B_3 + B_7 + B_9 \ . \tag{27}$$

Here the lower indices show the dimensions of the condensates, contained in the corresponding terms, A_0 is the contribution of the free quark loop.

We consider the radiative corrections to the terms A_0, A_6, and B_3. The radiative corrections to the other terms are not included since the values of the corresponding condensates are known with poor accuracy. We find, following [18], for the corresponding contributions from α_s-corrections to the Borel transformed SR:

$$\begin{aligned} \tilde{A}_0(M^2,W^2) = {} & M^6 E_2\left[1+\frac{\alpha_s}{\pi}\left(\frac{53}{12}-\ln\frac{W^2}{\nu^2}\right)\right] \\ & -\frac{\alpha_s}{\pi}\left[M^4W^2\left(1+\frac{3W^2}{4M^2}\right)e^{-W^2/M^2} \right. \\ & \left. + M^6\mathcal{E}\left(-\mathrm{W}^2/\mathrm{M}^2\right)\right], \end{aligned}$$

$$\tilde{A}_6(M^2,W^2) = \frac{4}{3}a^2 \times \left[1-\frac{\alpha_s}{\pi}\left(\frac{5}{6}+\frac{1}{3}\left(\ln\frac{W^2}{\nu^2}+\mathcal{E}(-\mathrm{W}^2/\mathrm{M}^2)\right)\right)\right],$$
$$\tilde{B}_3(M^2,W^2) = 2aM^4E_1\left(1+\frac{3}{2}\frac{\alpha_s}{\pi}\right) \tag{28}$$

with $\mathcal{E}(x)=\sum_{n=1}x^n/(n\cdot n!)$. Some terms in (28) differ from those in [18]. The numerical difference is, however, not very important.

The parts of equations proportional to α_s are the α_s-corrections to main terms. Now we compare the nucleon parameters obtained as solutions of nucleon SR without α_s-corrections,

$$m = 0.930 \text{ GeV}, \lambda^2 = 1.79 \text{ GeV}^6, W^2 = 2.00 \text{ GeV}^2,$$

and with α_s-corrections,

$$m = 0.94 \text{ GeV }, \lambda^2 = 2.00 \text{ GeV}^6, W^2 = 1.90 \text{ GeV}^2.$$

These results are presented for α_s=0.35. We show that in vacuum the radiative corrections modify mainly the values of the nucleon residue, while that of the nucleon mass suffers minor changes.

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